AD-A062 077 MINNESOTA UNIV MINNEAPOLIS F/G 20/4

STUDIES IN THE VISCOMETRY OF SLOW MOTIONS OF RHEOLOGICALLY COMP--ETC(U)

OCT 78 D D JOSEPH, G S BEAVERS DAAG29-76-G-0047

UNCLASSIFIED ARO-12754.16-E NL

1 OF 1

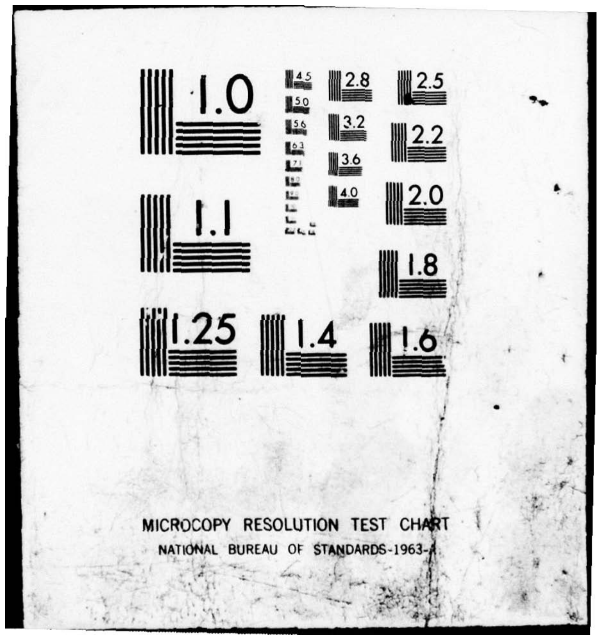
AD A062077

END

DATE FILMED

3--79

DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(18) Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(12)

ARO REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (19) 12754.16-E	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) Studies in the Viscometry of Slow Motions of Rheologically Complex Liquids.		5. TYPE OF REPORT & PERIOD COVERED (9) Final Report. 1 Oct 75 - 30 Sep 78
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) D. D. Joseph, G. S. Beavers		8. CONTRACT OR GRANT NUMBER(s) (15) DAAG29-76-G-0047
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Minnesota, Minneapolis, Minnesota 55455		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office, P. O. Box 12211, Research Triangle Park, NC 27709	(11) 31	12. REPORT DATE Oct[redacted]78
		13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 21p.	LEVEL	15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Flow Studies	Rheologically Complex Liquids
Complex Liquids	Viscometry
Weisenberg Problem	Non-Newtonian Fluids
Stoke's Flow Problem	Viscometric Data Characterization
Fluid Flow	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report summarizes the research accomplishments. The research has covered a broad range of problems associated with the viscometry of non-Newtonian fluids, from both analytical and experimental points of view. This report restates the objectives, reviews the major findings, and lists the abstracts of all the publications that have evolved from this research program.

78 12 04 016

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE 233 500

Unclassified

STUDIES IN THE VISCOMETRY OF SLOW MOTIONS OF RHEOLOGICALLY COMPLEX LIQUIDS

FINAL REPORT

BY

D. D. Joseph AND G. S. Beavers

Period Covered by Report: October 1, 1975 - September 30, 1978

U. S. ARMY RESEARCH OFFICE

Proposal No: P-12754-E Grant No: DAAG29-76-G-0047

University of Minnesota

Minneapolis, MN 55455

October 31, 1978

TABLE OF CONTENTS

PAGE

1. INTRODUCTION

This report summarizes the research accomplishments of the research program supported under U.S. Army Research Office Grant Number DAAG 29-76-G-0047. The research has covered a broad range of problems associated with the viscometry of non-Newtonian fluids, from both analytical and experimental points of view. In this report we restate our objectives, review the major findings of our work, and list the abstracts of all the publications that have evolved from this research program.

2. STATEMENT OF OBJECTIVES

The objectives of the research were outlined in the original proposal for this grant, and are repeated below:

> "Molten plastics and other rheologically complex fluids are increasingly used in modern technologies. Understanding the mechanics of flow of these fluids is important in applications ranging from the control of size and surface finish of plastic extrudates (for example, broom bristles and magnetic tapes) to the use of polymer additives (like STP) to improve and change the properties of common lubricating oils. Flow studies of rheologically complex fluids are intellectually attractive because of the variety of new and unexpected effects which are still being found in experiments, and because of the challenge to mathematical science which these effects present. Such studies have an unusual potential for direct, immediate and useful application.
>
> We seek support for experimental and mathematical studies of the mechanics of flow of rheologically complex liquids. The immediate aim of our studies is to enrich the science and technology of viscometry; we are trying to develop sets of standard experiments, founded on sound mathematical analysis, which will lead to reliable viscometric data characterizing the slow motion of rheologically complex fluids. We are also interested in certain mathematical studies of the mechanical foundations of rheology and in the evolution of new methods of analysis."

3. SUMMARY OF RESEARCH RESULTS

In this section we review briefly our research findings during the current grant period. We present our results under several headings, giving a few general remarks about each area. In the following section we list the publications which have originated in each area of research, and we summarize the results by means of the abstract of each publication.

3.1 Weissenberg Problems

Following our original work on the Weissenberg effect and our associated work on the rotating rod viscometer, we have completed work on several other Weissenberg problems.

3.1(a) Oscillating Rods

We have developed a general algorithm for computing small amplitude unsteady flows of viscoelastic (simple) fluids without presuming overly specific forms for the constitutive expressions relating stress and deformation. The general theory has been applied to the problem of finding the free surface between cylinders undergoing torsional oscillations, (Ref. 1). When a small rod rotates in a sea of viscoelastic fluid with an angular frequency equal to $\varepsilon \sin \omega t$, the amplitude is $\varepsilon = \omega\Theta/2$ where Θ is the angle of twist, (the maximum angular displacement of the rod). The theory has been applied to the experimental determination of the material functions which characterize the motions of viscoelastic fluids which have small amplitudes but are otherwise arbitrary, (Ref. 2). It is shown, in agreement with the theory, that when ε is small the time-averaged height rise at the rod divided by the angle of twist is a universal function of ω. This universal function may be used to determine the form of the two material functions which arise in the analysis. Once these functions are determined for a certain fluid the response of that same fluid may be predicted in any other motion in which the amplitude of the motion is small.

This research project was continued, and a more detailed experimental investigation of the fluid motion induced by a circular rod oscillating about its axis in a large volume of viscoelastic fluid has just been completed. The results of this work are contained in the M.S. Thesis of B. Kolpin. This work included a preliminary investigation of the effect of shear heating on the temperature of the fluid for different oscillation rates. These experiments allowed us to devise a procedure whereby the height-of-climb measurements could be made at a constant temperature. The agreement between theory and experiment was extended to higher values of the oscillation rate than obtained in previous experiments by extending the generalized Maxwell model to higher order. This work is now being prepared for publication (Ref. 19).

The analysis of Ref. 1 has been extended to include the configuration of oscillating parallel planes (Ref. 3).

3.1(b) Higher Order Theory for Motion Between Concentric Cylinders

The higher order theory of the Weissenberg effect has been completed, and appears in J. Yoo's Ph.D. Thesis and in Ref. 4. The theory shows that the values of the viscoelastic parameters through order four can be obtained by comparing theory and experimental rod-climbing results. In particular, one of the viscoelastic parameters can be obtained from direct measurements of the azimuthal component of velocity on the free surface near a rod rotating in a large volume of fluid. Extensive azimuthal velocity measurements have been made for several fluids, and show remarkable agreement with theory. The higher order theory predicts that at low angular speeds there is a secondary motion, driven by the vertical stratification of normal stresses and inertia, which drives the fluid in toward the rod, up the bubble and down the rod. At larger angular velocities the fluid falls down the big bubble and is entrained by the big eddy, setting up a double eddy system with a little eddy in the climbing head driven by the big eddy. These predictions have been observed in our experiments.

The higher order theory can also be used to predict the shape of the free surface on a fluid between concentric rotating cylinders. The shape depends upon the fluid, the diameters of the cylinders, and the speed of rotation. For some combinations of fluid and geometry the fluid has the greatest climb near the inner cylinder. For some other combinations the fluid climbs in the center of the gap and sinks near the inner and outer cylinders. We have demonstrated these effects by means of experiments with STP, TLA-227 and Polyacrylamide in glycerin-water mixture. The agreement between experimentally-measured profiles and predicted surface profiles is excellent. This work is described in detail in Ref. 20, which is now being prepared for publication. Some of the results are also summarized in our recent review article (Ref. 18).

3.1(c) Normal Stress Amplifier

We have continued research on our "normal stress amplifier," (Ref. 5). For example, we rotated a small circular rod in a vat with three fluid layers, air on top of STP on top of TLA 227. The Weissenberg effect at the STP - TLA 227 interface is magnified perhaps a hundred-fold by the small difference (0.005 gm/cm^3) in the density of the two non-Newtonian fluids. This effect is spectacular. We have explained this effect theoretically. The height of climb is roughly proportional to $1/\sqrt{\delta\rho}$ ($\delta\rho$ is the density difference) when the climb is not too large, and to $1/\delta\rho$ when the climb is very large. The "normal stress amplifier" uses the principle that the height of climb is a balance between normal stresses pushing up and gravity pulling down. By using fluids with the same density the gravity is turned off and normal stresses are unopposed. We believe that the normal stress amplifier has a potentially important application to the technology of material measurements for viscoelastic fluids. This aspect of our work has been presented in a recent issue of Scientific American (Jearl Walker, "The Amateur Scientist," Scientific American, Vol. 239, No. 5, November, 1978).

3.1(d) A Movie: "Novel Weissenberg Effects"

We have made a fifteen minute color movie on novel Weissenberg effects. (A copy of this movie has been deposited with the U.S. Army Research Office, Durham). The effects are peculiar to non-Newtonian fluids like STP. They include climbing on rotating rods, a Hopf bifurcation (the breathing instability) of the steady axisymmetric climb, the critical radius, the big effect of temperature and adhesion, a normal stress amplifier (how to remove gravity on earth), buckling of fluid towers, the mean climb on an oscillating rod and the symmetry breaking bifurcation of axisymmetric time-periodic flow (the flower instability).

3.2 Stokes' Flow Problems

We have developed a new type of solution for problems of Stokes flows in confined domains of simple shape. The method utilizes generalized Fourier series in biorthogonal functions. We have shown that these series always converge, and used them to compute secondary motions of viscoelastic fluids and in other problems previously thought to be intractable to analysis.

3.3 The Tilted Trough Problem

The tilted trough is used to measure the hard-to-measure second normal stress difference in viscoelastic fluids. Previous analysis of this problem made exaggerated claims about the applicability of the theory to experiments. We corrected these claims and extended the range of applicability of the theory. This work is described in Ref. 12.

3.4 Rotating Simple Materials

We derived new equations for studying arbitrary motions of viscoelastic fluids which perturb states of steady rigid rotation. At each stage of the perturbation one solves integral-differential equations for three components of velocity and a pressure. The problem of computing particle paths and velocities decouples, reducing the equations and unknowns from seven to four (as in the Navier-Stokes theory). The new theory applies to general viscoelastic fluids and not just to special models. The same material functions arise in the new theory as in our previous theory of perturbations of states of rest with arbitrary motions; previous theory is a special case of the new theory. We found that in some problems of rotating viscoelastic fluids there is an Ekman type of boundary layer which is much less thick than in Newtonian fluids. This work has been published as Ref. 13.

Recently this work has been extended to include general simple materials (i.e. both fluids and solids). The general theory is presented in Ref. 14.

3.5 Extrudate Swell

An analytical investigation of the "die-swell" problem has been initiated during the current grant period. The existence of an integral invariant of the axial momentum of the jet of liquid has been established. This work is described in Ref. 15.

A formal mathematical solution has been obtained for the very slow flow of a fluid issuing from a circular die. There remains to be shown certain aspects of the convergence of the solution as well as numerical computation of pertinent field quantities (i.e. velocities and stresses). With this slow flow solution, the jet free surface as it issues from the die can be obtained.

3.6 Deformation of Viscoelastic Solids

This project is the doctoral research of P-M Dixit. He has derived the form of the equations of motion of nonlinear viscoelastic solids in deformations which perturb the rest state. The rest state is assumed to be stress-free (natural) and isotropic. The material is assumed to obey the integral constitutive equation. The theory of nonlinear elasticity for static deformations results as a special case. The equations up to second order (for incompressible materials) involve two material constants and three material functions. He is attempting to solve the differential equations governing the displacements at second order for several simple geometries. These geometries will form the basis for experiments to determine some of the material constants and material functions.

3.7 Free Surface on a Fluid in a Cylinder with a Rotating Bottom

This project forms part of the doctoral research of A. Siginer. The aim is to determine some of the Rivlin-Ericksen coefficients through measurements of the free surface shape on a viscoelastic fluid in an open cylindrical container, the bottom of which rotates at constant angular speed. The mathematical problem reduces to the solution of a 4th-order problem in cylindrical coordinates. The method of solution involves the expansion of boundary data in biorthogonal series in terms of strip eigenfunctions. Since we do not have completeness theorems which pertain to those eigenfunctions, convergence of the series solution was checked by carrying out all the required computations. Unfortunately the data are non-canonical, and convergence was not reached. Other ways of solving the problem utilizing recently devised numerical simulation methods are being attempted.

3.8 Cusps in Fluids

We have observed that cusps can be generated in viscoelastic fluids when two moving fluid surfaces come into contact. We have started a program to study cusps in fluids, and this work forms part of the doctoral research of J. Sanders. To begin with we are looking at the problem of a cylinder rotating along its axis in the surface of a viscoelastic fluid. The cylinder is coated by the fluid, and a cusp forms at the free surface where the cylinder surface is moving down into the fluid. We are studying the coating flow problem for viscoelastic fluids as well as the cusp formation, from both experimental and analytical points of view.

4. PUBLICATIONS

The following publications have been produced on this grant.

4.1 Weissenberg Problems

Ref. 1 D. D. Joseph, "The free surface on a simple fluid between cylinders undergoing torsional oscillations. Part I; Theory," Archive for Rational Mechanics and Analysis, Vol. 62, No. 4, 323-342, 1976.

Ref. 2 G. S. Beavers, "The free surface on a simple fluid between cylinders undergoing torsional oscillations. Part II; Experiments," Archive for Rational Mechanics and Analysis, Vol. 62, No. 4, 343-352, 1976.

The combined abstract of Refs. 2 and 3 reads as follows:

"In a recent work (Joseph, 1976), ideas from the theory of domain perturbations were used to develop an algorithm for the computation of unsteady motions of a simple fluid. In this algorithm, the rest state is perturbed with an unsteady motion. The solution is expressed in powers of the amplitude ε of the unsteady data; the stress is expanded into a Fréchet series; the Fréchet stresses are represented as multiple integrals and the mutiple integrals are reduced to canonical form appropriate for the solution of practical problems of rheological fluid mechanics. In Part I of this paper we apply Joseph's algorithm to find the shape of the free surface on a simple fluid between cylinders undergoing torsional oscillations. In Part II we describe an experiment testing the predictions of Part I. Good agreement is demonstrated for all experimental tests of the theory when the amplitude of the oscillation is small."

Ref. 3 L. D. Sturges and D. D. Joseph, "The free surface on a simple fluid between cylinders undergoing torsional oscillations. Part III: oscillating planes," Archive for Rational Mechanics and Analysis, Vol. 64, No. 3, 245-267, 1977.

"In part I (Joseph, 1976A) of this three part paper, a recently developed algorithm (Joseph, 1976B) for computing the motions of a simple fluid of integral type which perturb the state of rest was applied to the problem of finding the shape of the free surface on a simple fluid between cylinders undergoing torsional oscillations. The analysis of this problem was carried out through terms of second order in the amplitude ε. When the cylinder radii are arbitrary, the solutions may be expressed in terms of Bessel functions but the resulting expressions are cumbersome. Simpler solutions are possible when the cylinders are infinitely far apart and when they are close together. In the first case, we are considering the change of elevation of the free surface on a sea of fluid around a rod undergoing torsional oscillations. In the second case, we are considering the change in elevation of the free surface on a fluid between oscillating parallel planes which oscillate with a velocity proportional to $\varepsilon \sin \omega t$. The first case was studied in Part I and the second case, here in Part III. The predictions following from the analysis of Part I were tested in the experiments reported in Part II (Beavers, 1976). The analysis of Part I could be criticized because the main formulas for the mean rise of height were based on an approximation, however good, and though a unique solution giving the secondary motion exists, it was not given. In the second limiting case, oscillating parallel planes, approximations are unnecessary and exact expressions are obtained for the mean rise in height, the oscillatory secondary flow and the oscillating change in the elevation of the free surface.

Four results of analysis appear to us to deserve early mention:

(i) The predictions of the response of all simple fluids of integral type in small-amplitude motions which perturb the rest state are completely determined when two material functions G(s) (the shear relaxation modulus) and $\gamma(s_1,s_2) = \gamma(s_2,s_1)$ (the quadratic relaxation modulus) are given. Various methods of determining G(s) and $\gamma(s_1,s_2)$ from experiments can be evaluated using this fact. For example, the same G(s) and $\gamma(s_1,s_2)$ should be determined by comparing theory and experiment for oscillating rods (Part I) and oscillating planes (Part III).

(ii) The mean rise in height of the fluid between oscillating planes is proportional to an unsteady equivalent $N_2(\omega)$ of the second normal stress. As $\omega \to 0$, $N_2(\omega) = 2\alpha_1 + \alpha_2$ where $2\alpha_1 + \alpha_2$ is the limiting value of the ratio of the second normal stress upon shear rate squared. Observations of the elevation changes on the oscillating free surfaces give rheological data which include information about the hard-to-measure second normal stress.

(iii) The mean rise between parallel planes is interesting because no rise occurs when planes are in steady motion and the shear rate is constant and uniform over the whole field of flow. In contrast, there is an appreciable steady rise on a rod rotating with even a small, steady, angular velocity. This contrast actually extends to the oscillating planes because the rise between planes is an order of magnitude smaller than the corresponding rise on the oscillating rod (compare H given by Fig. 8 of Part I and Fig. 6 of Part III). The big mean rise on the rod dominates the whole rise, but between oscillating planes the mean rise is much smaller and steady and unsteady changes in elevation can be equally important.

(iv) The solution of the fourth-order partial differential equation which governs the motion at second order is of a type which arises in the vibrations of elastic plates. We believe the nice eigenfunction solution which we construct is new and of potentially wide application."

Ref. 4 J. Y. Yoo, D. D. Joseph and G. S. Beavers, "Higher order theory of the Weissenberg effect," Journal of Fluid Mechanics, (in press).

"The higher order theory of the Weissenberg effect is developed as a perturbation of the state of rest. The theory is computed through order four where secondary motions first appear. Comparison of theory and experiment leads to the determination of five viscoelastic parameters. Some surprising effects are predicted and observed."

Ref. 5 G. S. Beavers and D. D. Joseph, "Novel Weissenberg Effects," Journal of Fluid Mechanics, Vol. 81, Part 2, 265-272, 1977.

"We have observed two novel manifestations of the Weissenberg effect in viscoelastic liquids which are set into motion by the rotation of a circular rod. In the first experiment we floated a layer of STP on water. The STP climbs up the rod into the air and down the rod into the water. The 'down-climb' is much larger than the 'up-climb', their ratio being roughly the square root of the density difference (STP-air)/(water-STP). The magnification of the down-climb may be regarded as normal-stress amplification. The magnitudes of the up- and down-climbs are simultaneously in good agreement with the predictions of a theory of rod climbing when the angular frequency of the rod is small. In the second experiment, we set the rod into torsional oscillations. When the amplitude of the oscillation is small, the fluid climbs the rod; the climb is divided into an axisymmetric steady mean part and an oscillating part (Joseph 1976b; Beavers 1976). The mean axisymmetric climb dominates the total climb at low frequencies. At a higher critical speed the axisymmetric climbing bubble loses its stability to another time-periodic motion with the same period but with a 'flower' pattern displaying a certain integral number of petals."

4.2 Stokes' Flow Problems

This method is developed and applied in the following papers:

Ref. 6 D. D. Joseph, "The convergence of biorthogonal series for biharmonic and Stokes flow edge problems. Part I," SIAM Journal of Applied Mathematics, Vol. 33, No. 2, 337-347, 1977.

"Sufficient conditions are established for the convergence of the biorthogonal series solving edge problems which arise in elasticity and in Stokes flow in cavities. These conditions greatly improve those stated in the excellent work of R. C. T. Smith (1952, Aust. J. Sci. Res. 5, 227)."

Ref. 7 L. Sturges and D. D. Joseph, "The convergence of biorthogonal series for biharmonic and Stokes flow edge problems. Part II," SIAM Journal of Applied Mathematics, Vol. 34, No. 1, 7-26, 1978.

"Sufficient conditions are established for the convergence of the biorthogonal series solving edge problems which arise in elasticity and in Stokes flow in cavities. These conditions and those given in Part I include all those which are likely to arise in applications. Examples of conditional convergence of the series to step functions and to ramp functions are presented. Problems previously considered to be intractable to analysis are solved by analysis."

Ref. 8 C. H. Liu and D. D. Joseph, "Stokes flow in wedge-shaped trenches," Journal of Fluid Mechanics, Vol. 80, Part 3, 443-463, 1977.

"In this paper we develop a separation of variables theory for solving problems of Stokes flow in wedge-shaped trenches bounded by radial lines and concentric circles centered at the vertex of the wedge. The theory leads to a set of Stokes flow eigenfunctions which in the full wedge reduce to the corner eigenfunctions studied by Dean & Montagnon (1949) and Moffatt (1964). Asymptotic formulae for the distribution of eigenvalues are derived, an adjoint system is defined and is used to develop an algorithm for the computation of the coefficients in an eigenfunction expansion of edge data prescribed on the circular boundaries. To illustrate the algorithm we find the motion and the shape of the free surface in a wedge-shaped cavity heated from its side."

Ref. 9 C. H. Liu and D. D. Joseph, "Stokes flow in conical trenches," SIAM Journal of Applied Mathematics, Vol. 34, No. 2, 286-296, 1978.

"In this paper we develop a separation of variables theory for solving problems of Stokes flow in cone-shaped trenches formed as the intersection of a cone of circular cross-section and a spherical shell centered at the vertex of the cone. The theory leads to a new set of Stokes flow eigenfunctions which describe axisymmetric motions in the vertex of a cone. Asymptotic formulas for the distribution of eigenvalues are derived; an adjoint system is defined and is used to develop an algorithm for the computation of the coefficients in an eigenfunction expansion of edge data prescribed on the spherical boundaries."

Ref. 10 J. Y. Yoo and D. D. Joseph, "Stokes flow in a trench between concentric cylinders," SIAM Journal of Applied Mathematics, Vol. 34, No. 2, 247-285, 1978.

"In this paper we develop a separation of variables theory for solving problems of Stokes flow in annular trenches bounded by horizontal parallel planes and concentric vertical cylinders. The theory leads to a new set of Stokes flow eigenfunctions, adjoint eigenfunctions, biorthogonality conditions and an algorithm for the computation of the coefficients in an eigenfunction expansion of edge data prescribed on the horizontal boundaries. To illustrate the algorithm we compute the motion and the shape of the free surface on a liquid in the space between two cylinders which are maintained at different temperatures. The solution is constructed as a domain perturbation of the rest state in powers of the temperature difference. At the lowest significant order the problem is reduced to a Stokes flow problem of the desired type with edge data prescribed on the horizontal boundaries."

Ref. 11 D. D. Joseph, "A new separation of variables theory for problems of Stokes flow and elasticity," Proceedings of the Second Symposium on 'Trends in Applications of Pure Mathematics to Mechanics', Kozubnik, Poland, 1977. Proceedings published by Pitman Publishing, London, 1978.

"Some classes of fourth order boundary-value problems arising in the theory of Stokes flow and elasticity are solved by the method of biorthogonal series. The eigenfunctions are formed from separable solutions when the separation constants (eigenvalues) are chosen to make the solution and its normal derivative vanish at the side-walls. A general and unified algorithm is presented for solving such problems on strips, in wedges, between disks, in cylinders and between cylinders and in cones. The method applies to many different equations and it always leads to the same general algorithms, the same type of biorthogonal expansion, the same type of reduction to ordinary differential equations, the same biorthogonality condition and the same type of formulas for the biorthogonal coefficients. The method leads to a new theory of biorthogonal "Fourier" series of two-component vector-valued functions. Convergence of the series is proved for sufficiently smooth but otherwise arbitrary data. Completeness of the representations is established in a smaller, but still large, class of functions. Questions of summability by Fejers method, Gibbs phenomenon, representation of functions in weak classes and other points of analysis in the theory of trigonometric series are important open questions in this new theory."

4.3 The Tilted Trough Problem

Ref. 12 L. Sturges and D. D. Joseph, "Slow motion and viscometric motion. Part V: the free surface on a simple fluid flowing down a tilted trough," Archive for Rational Mechanics and Analysis, Vol. 59, No. 4, 359-387, 1975.

"This paper is a contribution to the theory of viscometry of slow steady motions of a simple fluid and is presented as Part V of the work on slow motion and viscometric motion which formed the subject of the paper in four parts of JOSEPH (1974). The line of thought explored in all five parts, and in parts presently under preparation, is as follows: the constitutive relation for simple fluids is general enough to describe all motions of many fluids. The specification of the constitutive relation for a single fluid, the "name" of the fluid, is the general problem of viscometry. The very generality which is required to describe all of the possible responses of all simple fluids makes the solution of the general problem of viscometry difficult, to say the least. To circumvent this difficulty, we restrict our consideration to special motions on which the constitutive relation reduces to a manageable form; that is, a form for which it is possible to do the experiments to find the values of the quantities which name the fluid. We say that these manageable restrictions of the motion define restricted problems of viscometry (see Part I, Section 2). The two examples of restricted problems of viscometry under consideration are the viscometry of simple fluids in viscometric motions and the viscometry of simple fluids in slow steady motions. We are attempting to formulate a practically useful theory of viscometry for slow steady motions; this comes down to finding good relations from analysis to guide experiments to measure the Rivlin-Ericksen (RE, for short) constants which appear in the expressions (3.2) of Part I for the extra stress in slow steady flow.

The tilted trough viscometer, studied here, exploits the fact that the free surface on a viscoelastic fluid flowing down an open channel which is inclined to the horizontal will deform under the action of normal stresses induced by flow. The trough is a companion to the rotating rod viscometer (JOSEPH, BEAVERS & FOSDICK, 1974; BEAVERS & JOSEPH, 1975). Both the rod and the trough utilize the idea that the shape of the free surface depends on material parameters. By using the two instruments we can, at a minimum, determine the values of the two Rivlin-Ericksen constants of the 2nd order approximation relating stress to deformation in the slow steady motion of simple fluid.

We propose to use the formulas derived here as the guiding theory for the experimental determination of the special combinations of RE constants which arise in the study of flow down a tilted trough. We calculate the solution in a series of powers of the tilt angle β and show that secondary motions do not appear until sixth order when the trough is infinitely deep or has a semi-circular cross-section. For these troughs we give simple formulas relating the shape of the free surface to the RE constants of the first, second, third and fourth-order fluids."

4.4 Rotating Simple Materials

Ref. 13 D. D. Joseph, "Rotating simple fluids," Archive for Rational Mechanics and Analysis, Vol. 66, No. 4, 311-344, 1977.

"The canonical forms of the stress and the equations of motion governing arbitrary time-dependent perturbations of steady rigid rotation of general viscoelastic fluids are developed. The perturbation stresses are represented by integrals and the velocity, strain-history and pressure are developed in powers of a small parameter representing the perturbation of data which gives rise to rotation. The analysis shows that the strain history, which is computed from particle paths, may be computed as an auxilliary quantity after solving integral-differential equations for the three components of velocity and the pressure. The integral differential equations have a leading operator, to be inverted at each stage of the perturbation, the type encountered in the linear theory of viscoelasticity except that the active variables are velocity components and a pressure (as in the Navier-Stokes theory). The history of the velocity, which appears in the integral terms of the equations of motion, is evaluated on the past position of a particle in steady rigid rotation. The perturbed stresses thus have the axial symmetry of particles in steady rigid rotation and the position variables under the stress integrals are the same as the present position, except that the angular position of a particle presently at θ was $\theta - \Omega s$ where Ω is the angular velocity and s is the time lag. When $\Omega = 0$ the new theory reduces to Joseph's theory of perturbations of the state of rest; exactly the same material functions are needed in both theories. It is shown that various theories of rotating viscoelastic fluids by other authors are special cases of this one. It is shown that rigid rotation is stable when the shear relaxation modulus is of a physically realistic type and that the solution of the perturbation problems is unique under the same circumstance. Several problems are solved and considered in the limit $\Omega \to \infty$. For these problems one finds inertial stiffening of Ekman type boundary layers of a type familiar in the theory of rotating Newtonian fluids. The boundary layer in the viscoelastic fluid is much smaller than the boundary layer in a Newtonian fluid."

Ref. 14 D. D. Joseph, "Rotating simple materials," Journal of Non-Newtonian Fluid Mechanics, (in press).

"Equations of motion are derived for simple materials undergoing arbitrary small perturbations of the data giving rise to states of rest and rigid rotation. The fluid equations form the basis for a theory of rotating simple fluids with an associated boundary layer theory (analogous to Ekman's) and a theory of viscoelastic waves. Previous asymptotic results appear as special cases which arise from the new equations when the given data is of the right form. Differences in the fluid and solid equations and implications of the equations for experimental rheometry are discussed."

4.5 Extrudate Swell

Ref. 15 D. D. Joseph, "An integral invariant for jets of liquid into air," Archive for Rational Mechanics and Analysis, (in press).

"A liquid is forced to move from left to right (x increasing) down a round pipe of length L by high pressure imposed at the entrance $x = -L$ of the pipe. The flow is assumed to be axisymmetric but the pressure and velocity which is prescribed at $x = -L$ is otherwise arbitrary. At $x = 0$ the liquid is extruded into a zero gravity field and an axially-symmetric capillary jet of radius $h(x)$ is formed. I am going to prove the existence of an integral invariant of the axial momentum of the jet. This invariant is a locally conserved quantity which can be manipulated to produce the equation governing the global conservation of the axial momentum of the jet (see Part III, Joseph, 1974). The invariant should be useful in the analysis of die swell."

4.6 Review Papers

Ref. 16 D. D. Joseph and G. S. Beavers, "Free surface problems in rheological fluid mechanics," Rheologica Acta, Vol. 16, No. 2, 169-189, 1977.

Ref. 17 D. D. Joseph and G. S. Beavers, "Free surfaces induced by the motion of viscoelastic fluids," in The Mechanics of Viscoelastic Fluids, ed. R. S. Rivlin, ASME, AMD-22, 1977.

The two papers have essentially the same content which is summarized in the following abstract:

"In this paper we review our work on the theory of domain perturbations of the rest state of a viscoelastic fluid and its applications to the science of rheometry. To explain the principle behind the domain perturbation analysis we begin the paper with a discussion of a model problem, free of rheological complications, in which the principles involved may be clearly demonstrated. In Chapter two we formulate the analysis for rheological problems and we present some new, previously unpublished, simplifications of the theory. The canonical forms for stress tensors which perturb the state of rest are given in Chapter three. Chapter four is devoted to the problem of steady rod climbing. There we discuss the main physical features of the motion, the phenomenon of the critical radius, the effects of surface tension and temperature, secondary motions, the applicability of theory and experiment for rheological measurements, and an interesting new "normal stress amplifier". Unsteady problems of rod climbing are considered in Chapter five. We review recent results on the free surface induced by torsional oscillations of a rod, and we show how to use these results to find the form of the material functions in the canonical forms of the stress. The breathing instability of steady axisymmetric rod climbing (a Hopf bifurcation) and the flower instability of time-periodic climb induced by the oscillating rod (a symmetry-breaking Poincare bifurcation) are also described in Chapter five. In Chapter six we consider other free surface problems: the free surface on a fluid between oscillating planes, die swell and edge effects in rheometers."

Ref. 18 G. S. Beavers and D. D. Joseph, "Experiments on free surface phenomena," Journal of Non-Newtonian Fluid Mechanics (in press).

"This paper reviews an experimental program in which predictions from domain perturbation theory for motions which perturb the rest state are used in conjunction with experimental measurements on free surface deformations associated with the Weissenberg effect at low rates of shear to obtain values for certain rheological parameters. The experiments include free surface measurements on a liquid near a circular rod rotating in steady motion and oscillatory motion in a large volume of fluid, secondary flows and circumferential velocity measurements in steady rod climbing, normal stress amplification effects, and free surface shapes on a fluid confined between rotating cylinders. Temperature effects are noted, and interesting instability phenomena at high rates of shearing are demonstrated."

4.7 Papers in Preparation

Ref. 19 B. E. D. Kolpin, G. S. Beavers and D. D. Joseph, "The Free Surface on a Simple Fluid Between Cylinders Undergoing Torsional Oscillations. Part IV: Oscillating Rods."

Ref. 20 J. Y. Yoo, G. S. Beavers and D. D. Joseph, "The Free Surface on a Liquid Between Cylinders Rotating at Different Speeds: Part III."

5. PARTICIPATING PERSONNEL

The following personnel have participated in this research during the three-year tenure of the grant:

Professor D. D. Joseph	- Principal investigator
Professor G. S. Beavers	- Principal investigator
P-M Dixit	- Research assistant
Barbara E. D. Kolpin	- Research assistant
J. Sanders	- Research assistant
A. Siginer	- Research assistant
L. Sturges	- Research assistant
S. Trogdon	- Research assistant
J. Y. Yoo	- Research assistant

6. EDUCATIONAL ACHIEVEMENTS AND INVITED PRESENTATIONS

In this section we list some of the major achievements related to the research carried out as part of the current program.

6.1 Educational

The following theses have been completed on this research program:

L. D. Sturges, Ph.D. 1977. Thesis title: "Studies in the slow flows of viscoelastic fluids."

J. Y. Yoo, Ph.D. 1977. Thesis title: "Fluid motion between two cylinders rotating at different speeds."

B. E. D. Kolpin, M.S. 1977. Thesis title: "The oscillating rod viscometer."

The following students are presently working towards advanced degrees on this program:

P. M. Dixit, Ph.D. candidate

J. Sanders, Ph.D. candidate

A. Siginer, Ph.D. candidate

S. Trogdon, Ph.D. candidate

6.2 Invited Presentations

The following is a list of the major invited lectures which cover our work on this research program:

D. D. Joseph: "Rigorous theory of perturbation of the rest state of a simple fluid." Euromech Colloquium No. 79 on "Solutions to Basic Problems in Non-Linear Continua," Darmstadt, West Germany, September 7-10, 1976.

D. D. Joseph: "A new separation of variables theory for problems of Stokes flow and elasticity." Second Symposium on "Trends in Applications of Pure Mathematics to Mechanics," Kozubnik, Poland, September 12-17, 1977.

D. D. Joseph: "Constitutive equation and free surfaces." Symposium on Continuum Mechanics and Partial Differential Equations, Rio de Janeiro, Brazil, August 1-5, 1977.

D. D. Joseph and G. S. Beavers: "Free surfaces induced by the motion of viscoelastic fluids." Symposium on "The Mechanics of Viscoelastic Fluids," 1977 Joint Applied Mechanics, Fluids Engineering, and Bioengineering Conference, Yale University, New Haven, June 15-17, 1977.

D. D. Joseph: "Rotating simple materials." IUTAM Symposium on "Non-Newtonian Fluid Mechanics," Louvain-la-Neuve, Belgium, August 28 - September 1, 1978.

G. S. Beavers: "Experiments on free surface phenomena." IUTAM Symposium on "Non-Newtonian Fluid Mechanics," Louvain-la-Neuve, Belgium, August 28 - September 1, 1978.